17-3 The Poisson Distribution

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The Poisson Distribution arises when you count a number of events across time or over an area.

For example:

The number of phone calls received during a given period of time

The number of car accidents at a certain location during a given period of time

The number of particles emitted by a radioactive source in a given time

The number of fish caught in a survey net in a very large lake on a given day.

The number of criminal events that Spiderman needs to respond to over the course of a 24 hour period.

Assume that the interval is divided into a very large number of subintervals so that the probability of occurrence of an event in any sub-interval is very small. The term “interval” refers to either a fixed time interval of a fixed area. The Poisson Distributiion is based on these 4 assumptions:

1.) The probability of observing a single event over a small interval is approximately proportional to the size of the interval.

2.) The probability of 2 events occurring in the same narrow interval is negligible.

3.) The probability of an event in one interval does not change over different intervals.

4.) The probability of an event in one interval is independent of the probability of an event in a non-overlapping interval.

The Poisson Distribution

μ = The average number of events observed over a specific interval.

X = The number of events we are interested in (# of events you want to observe). Must be a positive integer.

$$P(X = x) = \frac{e^{-\mu}\mu^{x}}{x!}$$

with expected value

$E(x) = \mu$

Ex1. Recordable accidents occur in a factory at a an average rate of 7 every year, independently of each other. Find:

a.) The probability that in a given year, exactly 3 recordable accidents occurred.

b.) The probability that in a given year, exactly 7 recordable accidents occurred.

c.) The probability that in a given year, exactly 10 recordable accidents occurred.

a.) $\mu = 7 \quad x = 3$

$$P = \frac{\mu^x e^{-\mu}}{x!}$$

$$P = \frac{7^3 e^{-7}}{3!} \approx .0521$$

5.21%

b.) $$P(X=7) = \frac{e^{-7} 7^7}{7!}$$

$\approx .1490$

c.) $$P(x=10) = \frac{e^{-7} \cdot 7^{10}}{10!}$$

$= .0710$

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Cumulative Probability Distribution

$$P(X \le x) = \sum_{i=0}^{x} \frac{e^{-\mu}\mu^{i}}{i!}$$

d.) The probability of there being less than 5 recordable accidents in a year.

$M = 7 \quad x < 5$

poissoncdf(7, 5)

$= .1730$

e.) The probability of there being 8 or more accidents in a given year.

$P(x \geq 8) = 1 - P(X \leq 7)$

$= 1 -$ poissoncdf(7, 7)

$= .401$

Ex3. From a particular observatory, shooting stars are observed in the night sky at an average rate of one every 5 minutes. Assuming that this rate is constant and that shooting stars occur (and are observed) independently of each other, what is the probability that 20 or more are seen over a period of 1 hour?

$\mu = 12/hr$

$x = 20/hr$

$$P = 1 - P(x \le 19)$$

$$= 1 - \text{Poissoncdf}(12, 19)$$

$.0213$

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NORMAL FLOAT AUTO REAL RADIAN MP
poissoncdf(12,19)
                    .9787202307
1-Ans
                    .0212797693
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Ex4. When examining blood from a healthy individual under a microscope, a haematologist knows that on average he should see 4 white blood cells in each high power field. Find the probability that blood from a healthy individual will show:

a.) 7 white blood cells in a single high power field.

$$P(x=7) = \frac{e^{-4} \cdot 4^7}{7!} \approx .0595$$

b.) A total of 28 white blood cells in 6 high power fields, selected independently.

$x = 28$ $\mu = 4 \cdot 6 = 24$

$P = .0548$

NORMAL FLOAT AUTO REAL RADIAN MP

poissonpdf(24,28)
.0547906408

Ex4. Patients arrive at random at an emergency room in a hospital at the rate of 14 per hour throughout the day.

a.) Find the probability that exactly 4 patients will arrive at the emergency room between 18:00 and 18:15.

$\mu = \frac{14}{4} = 3.5$

$P(X=4) = \frac{e^{-3.5} \cdot 3.5^4}{4!}$

$= .189$

b.) Given that fewer than 15 patients arrive in one hour, find the probability that more than 12 arrive.

$P(x > 12 / x < 15)$

$$\frac{P(x = 13 \text{ or } x = 14)}{P(x \le 14)} = \frac{\text{poissonpdf}(14,3) +}{\text{poissoncdf}(14,14)}$$

$\approx .372$

c.) Find the probability that exactly 14 patients come into the emergency room every hour for a 4 hour shift.

$$\left[P(X=14)\right]^4 = \left[\frac{e^{-14}\,14^{14}}{14!}\right]^4 \approx .000126$$

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$\mu = 14$

d.) Find the probability that in 4 consecutive hours, fewer than half of them have any patients.

Case #1

4 hrs no patients

case #2

3 hrs w/no patients

1 hrs w/some patients

$P(\text{no patients}) = P(x=0)$

$P(x=0) = \dfrac{e^{-14} \cdot 14^0}{0!} = 8.315 \times 10^{-7}$

Case #1

$P(x \neq 0) = 1 - \text{Ans} = .999992$

$(8.315 \times 10^{-7})^4 = 4.781 \times 10^{-25}$

Case #2

____ · ____ · ____ · ____

Binomial

$\binom{4}{3} \cdot (8.315 \times 10^{-7})^3 \cdot (.999992)^1$

$\approx 2.2996 \times 10^{-18}$

$\boxed{P = 2.2996 \times 10^{-18} + 4.78 \times 10^{-25}}$

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